The Structure and Circulation of the Deep Venus Atmosphere

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ABSTRACT

A simple model for the structure of a non-rotating Hadley regime in an atmosphere with large thermal inertia is developed. The radiative fluxes are estimated by using a linearization about the radiative equilibrium state and the dynamical fluxes are estimated by using scaling analysis. The requirement that differential heating by these fluxes be in balance in both the meridional and vertical directions leads to two equations for the mean static stability and meridional temperature contrast. The solution depends on two parameters: the strength of the radiative heating, as measured by the static stability A_e of the radiative equilibrium state; and the ratio of the time it takes an external gravity wave to traverse the atmosphere to the time it would take the atmosphere to cool off radiatively, denoted by ϵ.

In the deep Venus atmosphere $\epsilon \approx 10^{-6}$; the equations are therefore analyzed in the limit $\epsilon \rightarrow 0$. The large-scale dynamics has virtually the same effect on the lapse rate as small-scale convection: if $A_e>0$ the radiative lapse rate is unchanged, while if $A_e<0$ the lapse rate becomes subadiabatic, but only by an amount of order $\epsilon^{\frac{1}{3}}$. Therefore, one need not invoke convection to explain the approximate adiabatic lapse rate in the Venus atmosphere, but a greenhouse effect is necessary to explain the high surface temperatures. The other properties of the solutions when $A_e<0$ are consistent with observational evidence for the deep atmosphere: the horizontal velocities are typically $\sim$2 m sec^{-1}, the vertical velocities $\sim\frac{1}{2}$ cm sec^{-1}, and the meridional temperature contrast is unlikely to exceed 0.1K.

The same approach is used to study the time-dependent problem and determine how long it would take for a perturbed atmosphere to reach equilibrium. If $A_e>0$ the adjustment is primarily governed by the radiative time scale, which is about 100 earth years for the deep Venus atmosphere. If $A_e<0$ the adjustment is governed by an advective time scale which may be as short as 20 earth days. Published numerical studies of the deep circulation have only treated the first case, but their integrations were not carried beyond about 200 earth days and therefore do not describe true equilibrium states. Only the second case, $A_e<0$, is consistent with the observations and it would be relatively easy to study numerically.

1. Introduction

In the last seven years the Venera probes have obtained the first direct information about motions in the atmosphere of Venus (Kerzhanovich *et al.*, 1972; Marov *et al.*, 1973). However, the velocities measured by the probes show a variety of magnitudes and directions. For example, Venera 4 measured a maximum meridional velocity $\sim$50 m sec^{-1}, Venera 7 a maximum zonal velocity $\sim$10 m sec^{-1}, and Venera 8 a maximum zonal velocity $\sim$100 m sec^{-1}. Evidently, a much larger number of probes covering the whole planet is needed to determine the mean circulation, and current discussions of the general circulation must rely on theoretical investigations.

The probes do supply indirect information about the general circulation, through measurements of quantities other than winds. For example, Venera 8 measured the solar shortwave flux as a function of depth in the atmosphere (Avduevsky *et al.*, 1973) and found that about 5% penetrates to the lowest scale height. Since the differential solar heating drive for atmospheric motions is proportional to the one-fourth power of the absorbed flux, this result strongly suggests that motions and dynamical fluxes will be important throughout the atmosphere. Also the probes have shown that the lapse rate throughout the deep atmosphere is very close to adiabatic (Marov *et al.*). This result is a constraint on any theoretical investigation.

The first theoretical discussion of the deep circulation was presented by Goody and Robinson (1966). Their basic hypothesis was that, in an atmosphere with negligible rotation subject to differential solar heating, the general circulation would consist of a simple cellular overturning, with rising motions in regions of net heating and sinking motions in regions of net cooling. This kind of motion is generally referred to as a Hadley cell. Since their hypothesis is essentially a statement that the motions will be in a thermodynamically direct sense, and since such motions are observed in many analogous situations [see Stone (1968) for references], it is difficult to argue that the general circulation will not be a Hadley cell, at least in an average sense. In fact, all subsequent discussions of the general circulation of the deep atmosphere have explicitly or implicitly adopted this hypothesis, and we will do so in this paper.

Some of the other points in Goody's and Robinson's original discussion have to be modified in the light of subsequent results. For example, it is now clear that the thermal inertia of the deep atmosphere is so large (Thaddeus, 1968) that the thermal drives must be primarily meridional rather than zonal. Also the Venera 8 measurement of a significant shortwave flux penetrating to all levels of the atmosphere indicates that thermal boundary layers are not likely to dominate the flow patterns. In addition, it is clear that a thermodynamically direct circulation cannot heat the surface of Venus. By definition, warm air rises and cool air sinks, so that the net vertical flux of heat across any level surface is upward, and the circulations must cool the ground. Thus, the most plausible explanation for the high surface temperatures is the greenhouse effect (Sagan, 1962; Pollack, 1969), particularly in view of the Venera 8 flux measurements.

Subsequent theoretical investigations of the properties of the general circulation fall into two classes. The first class avoids solving the equations of motion and makes simplifying assumptions in order to estimate the magnitude of important parameters. The scaling analyses of Goody and Robinson (1966), Stone (1968) and Gierasch *et al.* (1970), and the similarity analysis by Golitsyn (1970) fall into this first class. The first two scaling analyses assumed that dynamical heating and cooling in the deep atmosphere are balanced by small-scale turbulent diffusion, while the third (Gierasch *et al.*) assumed that they are balanced by radiative heating and cooling. In view of the Venera 8 flux measurements, the latter assumption appears to be more realistic. Golitsyn's similarity analysis assumed that the only important external parameters were the amount of solar energy absorbed, the specific heat and mass of the atmosphere, the radius of the planet, and the Stefan-Boltzmann constant.

If we adopt the analysis by Gierasch *et al.* as the most plausible scaling analysis, and apply it to the lowest scale height of the Venus atmosphere, we deduce meridional velocities ~ 1 m sec^{-1} and a temperature contrast between the equator and the poles ~ 1K. Golitsyn's similarity analysis led to the same estimates for these two quantities. This agreement between two quite diverse approaches gives considerable credence to the estimates. The velocity estimate is consistent with the Venera measurements, which showed velocities in the lowest scale height of the order of a few meters per second or less. The larger velocities quoted above occur at altitudes near 50 km where atmospheric conditions differ considerably from the lowest scale height. In particular, at high altitudes the thermal time constants are much shorter and the circulations may be quite different from the deep circulation (Schubert and Young, 1970). Also, one would expect higher velocities at high altitudes simply because of mass continuity; if the deep circulation extends throughout the deep atmosphere, a 1 m sec^{-1} velocity near the surface would become a 100 m sec^{-1} velocity five scale heights above the surface. The variations in velocities found by the different Venera probes could be due to the general instability of shear flows on Venus to small-scale disturbances (Hart, 1972). The estimate of a small temperature contrast is also consistent with thermal maps (Murray *et al.*, 1963) showing only slight temperature contrasts near the cloud tops.

Another striking agreement between Gierasch *et al.*'s scaling analysis and Golitsyn's similarity analysis was the deduction of the same dimensionless parameter as that which controls the dynamics on Venus. This parameter is the ratio of a dynamical time scale to a radiative time scale. The dynamical time scale is the time required for an external gravity wave to traverse the planet, $\sim 3\times 10^4$ sec on Venus. The radiative time scale is the relaxation time required for the deep atmosphere to cool radiatively, $\sim 4\times 10^9$ sec on Venus.

The second class of theoretical investigations consists of detailed numerical solutions of the equations of motion and the energy equation. Such investigations have been presented by Hess (1968), Sasamori (1971), Turikov and Chalikov (1971) and de Rivas (1973). Detailed calculations like these are necessary for determining important quantities such as the depth of the Hadley cell and the strength of the vertical motions at the cloud levels. All these investigations took the same approach: an initial state was specified, the equations were integrated using a time-marching procedure, and when the solution appeared to have reached equilibrium the integration was stopped. In all of these studies the integrations were stopped after a time $\sim 2\times 10^7$ sec. As noted by de Rivas this time is short compared to the radiative relaxation time; it is therefore questionable whether these calculations have indeed attained an equilibrium state. They may have described only quasi-equilibrium states quite different from the mean state.

This possibility becomes more likely when one examines the behavior of the static stability in these integrations. In all cases the integrations started with a adiabatic lapse rate, and after 2×10^7 sec the lapse rate showed only small changes. Yet the deviations from the adiabatic lapse rate must be calculated accurately in order to describe the equilibrium dynamics. The small temperature contrasts observed and deduced theoretically require a poleward transport of heat by the large-scale circulations. This means that the poleward branch of the Hadley cell in the higher atmospheric levels must on average be at a higher potential temperature than the equatorward branch in the lower levels. Thus, the lapse rate at least in a average sense must be subadiabatic. If the lapse rate were exactly adiabatic there would be no poleward heat flux. The flux is proportional to the difference between the actual lapse rate and the adiabatic lapse rate, and this difference, even though it is very small judging from the Venera measurements, must be taken into account in any dynamical calculation.

In order to estimate the static stability in the lower atmosphere of Venus we present in this paper a simple extension of the scaling analysis given by Gierasch *et al.* (1970). In particular, we will relax their assumption that the horizontal and vertical contrasts of potential temperature are comparable. Two equations for these two contrasts may be obtained simply by writing conservation equations for both the vertical and horizontal energy fluxes, in place of the simple global energy balance equation used by Gierasch *et al.* Simultaneously, we will assess the sensitivity of the equilibrium state to the static stability, and the success of the published numerical integrations in simulating the equilibrium state.

2. The mathematical model

For our scaling analysis we will follow Gierasch *et al.* We assume that the motion is two-dimensional and steady; neglect rotation and curvature effects; assume hydrostatic equilibrium; model radiative heating qualitatively by the simple linearization developed by Spiegel (1957) and Goody (1964); and neglect all other small-scale transport processes. We will differ from Gierasch *et al.* in that we will not make the Boussinesq approximation. If the non-Boussinesq equations are written in pressure coordinates they are no more difficult to use in a scaling analysis than the Boussinesq equations, and they are of more general applicability. However, our qualitative results would not be changed by using the Boussinesq equations.

The equations expressing conservation of mass, momentum, and energy are then as follows:

$$\frac{\partial v}{\partial y}+\frac{\partial \omega}{\partial p}=0, \tag{2.1}$$

$$\frac{\partial}{\partial y}(v^2)+\frac{\partial}{\partial p}(\omega v)=-\frac{\partial \phi}{\partial y}, \tag{2.2}$$

$$\frac{\partial \phi}{\partial p}=-\frac{RT}{p}, \tag{2.3}$$

$$\frac{\partial}{\partial y}(v\theta)+\frac{\partial}{\partial p}(\omega\theta)=\frac{\theta_e-\theta}{\tau}, \tag{2.4}$$

where y is the meridional coordinate; p the pressure normalized so that the average surface pressure is $p=1$; v the meridional velocity; ω the time rate of change of p, i.e.,

$$\omega=\frac{dp}{dt}, \tag{2.5}$$

and is analogous to the vertical velocity; ϕ the geopotential height; R the gas constant; T the temperature; θ the potential temperature,

$$\theta=Tp^{(1-\gamma)/\gamma}; \tag{2.6}$$

γ the ratio of specific heats; θ_e the radiative equilibrium solution for θ and is a function of y and p; and τ the radiative relaxation time for perturbations with a vertical scale equal to the scale height.

We treat Eqs. (2.1)–(2.3) in the same way as Gierasch *et al.* For the deep circulation the scale of p is O(1) and the scale of y is O(L), where L is the equator-to-pole distance. Thus, in order of magnitude, Eqs. (2.1)–(2.3) require that

$$\omega=\mathrm{O}\left(\frac{v}{L}\right), \tag{2.7}$$

$$v^2=\mathrm{O}(\Delta\phi), \tag{2.8}$$

$$\Delta\phi=\mathrm{O}(R\Delta T), \tag{2.9}$$

where $\Delta\phi$ and ΔT are the magnitudes of the horizontal variations of ϕ and T. We use the horizontal variations for estimating magnitudes in Eqs. (2.2) and (2.3) since it is the horizontal variations of geopotential height arising from horizontal temperature gradients which drive the large-scale motions. The magnitude of the horizontal variations of potential temperature, $\Delta\theta$, follows from Eq. (2.6):

$$\Delta\theta=\mathrm{O}(\Delta T). \tag{2.10}$$

We will consider as our prime unknowns the dimensionless mean gradients of the potential temperature,

$$A\equiv-\frac{1}{\bar\theta}\frac{\partial\bar\theta}{\partial p}, \tag{2.11}$$

$$B\equiv-\frac{L}{\bar\theta}\frac{\partial\bar\theta}{\partial y}=\mathrm{O}\left(\frac{\Delta\theta}{\bar\theta}\right), \tag{2.12}$$

where the bar indicates an average over all y and p. In the above A is a measure of the mean static stability of the atmosphere, and B is essentially the equator-to-pole temperature contrast normalized by the mean temperature. We can relate the velocity magnitudes to B by using Eqs. (2.7)–(2.10) and (2.12):

$$v=\mathrm{O}[(R\bar\theta B)^{\frac{1}{2}}], \tag{2.13}$$

$$\omega=\mathrm{O}\left[\frac{(R\bar\theta B)^{\frac{1}{2}}}{L}\right]. \tag{2.14}$$

Therefore, $B^{\frac{1}{2}}$ gives the magnitude of the horizontal velocity, normalized to the phase speed of an external gravity wave.

Gierasch *et al.* assumed that $A=\mathrm{O}(B)$, and derived an order-of-magnitude equation for B from (2.4). We will not make this assumption, but will derive two order-of-magnitude equations for A and B from (2.4). For boundary conditions we will require that the motions go to zero at the boundaries of the Hadley cell:

$$v=0 \quad \text{at} \quad y=0, L, \tag{2.15}$$

$$\omega=0 \quad \text{at} \quad p=p_s, p_t. \tag{2.16}$$

We will consider the Northern Hemisphere Hadley cell, so that $y=0$ corresponds to the equator and $y=L$ to the North Pole. Therefore, B is a positive definite quantity. The surface pressure is p_s, with a mean value equal to unity because of our scaling, and p_t is the pressure at the top of the regions where significant absorption of solar radiation occurs ($p_t \ll 1$). If we integrate Eq. (2.4) over the whole lower atmosphere, the above boundary conditions insure that

$$\bar{\theta}=\bar{\theta}_e, \tag{2.17}$$

i.e., the mean potential temperature is only affected by radiative transfer, and the dynamics only redistributes the thermal energy.

To obtain our first equation we integrate (2.4) over p, apply boundary conditions (2.16), evaluate the equation separately at the equator and the pole, and subtract the two resulting equations. We obtain

$$\left[\int_{p_s}^{p_t} \frac{\partial}{\partial y}(v\theta)dp\right]_0^L = \left[\int_{p_s}^{p_t} \frac{\theta_e-\theta}{\tau}dp\right]_0^L. \tag{2.18}$$

This equation states that net radiative cooling at the pole relative to the equator is balanced by dynamical heating. Our second equation is obtained by integrating (2.4) over y, applying boundary conditions (2.19), evaluating the equation separately at p_s and p_t, and subtracting the two resulting equations:

$$\left[\int_0^L \frac{\partial}{\partial p}(\omega\theta)dy\right]_{p_s}^{p_t} = \left[\int_0^L \frac{\theta_e-\theta}{\tau}dy\right]_{p_s}^{p_t}. \tag{2.19}$$

This equation states that net radiative heating high in the atmosphere relative to low in the atmosphere is balanced by dynamical cooling.

To evaluate the terms in Eqs. (2.18) and (2.19), we adopt as a simple representation of the potential temperature field, adequate for order-of-magnitude estimates, a linear function of y and p with gradients equal to the mean values, i.e.,

$$\theta=\bar{\theta}\left[1-A\left(p-\frac{1}{2}\right)-B\left(\frac{y}{L}-\frac{1}{2}\right)\right]. \tag{2.20}$$

Substituting (2.20) into (2.6) we find for the corresponding heating function

$$\theta_e-\theta=\bar{\theta}\left[(A-A_e)\left(p-\frac{1}{2}\right)+(B-B_e)\left(\frac{y}{L}-\frac{1}{2}\right)\right]. \tag{2.21}$$

The subscript e always indicates quantities evaluated for the radiative equilibrium structure. Substituting (2.21) into the radiative terms in Eqs. (2.18) and (2.19), and approximating $p_s=1$, $p_t=0$, we find

$$\left[\int_{p_s}^{p_t} \frac{\theta_e-\theta}{\tau}dp\right]_0^L = \mathrm{O}\left[\frac{\bar{\theta}}{\tau}(B_e-B)\right], \tag{2.22}$$

$$\left[\int_0^L \frac{\theta_e-\theta}{\tau}dy\right]_{p_s}^{p_t} = \mathrm{O}\left[\frac{L\bar{\theta}}{\tau}(A_e-A)\right]. \tag{2.23}$$

We approximate the derivatives in the dynamical terms by using simple differences, applying boundary conditions (2.15) and (2.16), and again approximating $p_s=1$, $p_t=0$, to obtain

$$\left[\int_{p_s}^{p_t} \frac{\partial}{\partial y}(v\theta)dp\right]_0^L \approx -\frac{4}{L}\int_1^0 v\theta|_{y=\frac{1}{2}L}dp, \tag{2.24}$$

$$\left[\int_0^L \frac{\partial}{\partial p}(\omega\theta)dy\right]_{p_s}^{p_t} \approx 4\int_0^L \omega\theta|_{p=\frac{1}{2}}dp. \tag{2.25}$$

To estimate these integrals we will again use our expression (2.20) for θ, and we will also use simple linear functions for the velocities, i.e.,

$$v|_{y=\frac{1}{2}L}=(R\bar{\theta}B)^{\frac{1}{2}}(1-2p), \tag{2.26}$$

$$\omega|_{p=\frac{1}{2}}=\frac{(R\bar{\theta}B)^{\frac{1}{2}}}{L}\left(\frac{2y}{L}-1\right). \tag{2.27}$$

The constants in these two functions have been chosen so that mass is conserved (with $p_s=1$, $p_t=0$) and so that the velocity magnitudes agree with those given by the scaling analysis [Eqs. (2.13) and (2.14)]. Substituting (2.20), (2.26) and (2.27) into (2.24) and (2.25), we obtain

$$\left[\int_{p_s}^{p_t} \frac{\partial}{\partial y}(v\theta)dp\right]_0^L = \mathrm{O}\left[+\frac{A\bar{\theta}}{L}(R\bar{\theta}B)^{\frac{1}{2}}\right], \tag{2.28}$$

$$\left[\int_0^L \frac{\partial}{\partial p}(\omega\theta)dy\right]_{p_s}^{p_t} = \mathrm{O}\left[-B\bar{\theta}(R\bar{\theta}B)^{\frac{1}{2}}\right]. \tag{2.29}$$

Finally we substitute (2.22), (2.23), (2.28) and (2.29) into (2.18) and (2.19) to obtain two order-of-magnitude equations for A and B,

$$B-B_e=-\frac{1}{\epsilon}AB^{\frac{1}{2}}, \tag{2.30}$$

$$A-A_e=\frac{1}{\epsilon}B^{\frac{3}{2}}, \tag{2.31}$$

where

$$\epsilon=\frac{\tau_D}{\tau}, \tag{2.32}$$

$$\tau_D=\frac{L}{(R\bar{\theta})^{\frac{1}{2}}}. \tag{2.33}$$

In the above τ_D is the dynamical time scale referred to in the Introduction, i.e., the time it takes for an external gravity wave to propagate from the equator to the pole; and ϵ is the same dimensionless parameter that Gierasch *et al.* (1970) and Golitsyn (1970) found to be so important in describing a non-rotating atmosphere.

3. Equilibrium solutions

Gierasch *et al.* assumed that $A=O(B)$. If this assumption is introduced into Eq. (2.30), it reduces to their Eq. (7). However, the simultaneous solution of Eqs. (2.30) and (2.31) will not in general be such that $A=O(B)$. Also we note that the solution will, in fact, always have two properties which we noted in Section 1. Since B has been defined to be a positive quantity [cf. Eq. (2.12)], Eq. (2.30) requires that

$$B<B_e \quad \text{only if} \quad A>0, \tag{3.1}$$

i.e., the dynamical transports will act to reduce the horizontal temperature gradient only if the atmosphere is statistically stable. Also, referring to Eq. (2.31) we see that

$$A>A_e, \tag{3.2}$$

i.e., the dynamical transports always stabilize the atmosphere. Since $\bar{\theta}=\bar{\theta}_e$, this result assures us that the dynamics always cools the ground, on the average.

To solve Eqs. (2.30) and (2.31) for Venus we must specify appropriate values of B_e, A_e and ϵ. We can estimate B_e by assuming that the insolation varies as the cosine of the latitude ϕ so that the meridional variation of the radiative equilibrium solution is governed by

$$T_e(y,p)=T_e(p)\left(\frac{4}{\pi}\cos\phi\right)^{\frac{1}{4}}. \tag{3.3}$$

Substituting (2.6) and (3.1) into (2.12), and choosing $\phi=O(\frac{1}{4}\pi)$, $\bar{T}_e=O(\bar{\theta}_e)$, and $p=O(1)$ as typical, we find

$$B_e=\frac{\pi}{8}\left(\frac{4}{\pi}\right)^{\frac{1}{4}}p^{(1-\gamma)/\gamma}\frac{\bar{T}_e}{\bar{\theta}_e}\frac{\sin\phi}{(\cos\phi)^{\frac{3}{4}}}=O(1). \tag{3.4}$$

Our knowledge of the absorbing properties of the Venus atmosphere is not yet sufficient to allow us to estimate A_e. Consequently, we will leave A_e as a parameter and find solutions as a function of A_e. However it is useful to note that for an isothermal atmosphere

$$A_e=\frac{R}{c_p}p^{(1-2\gamma)/\gamma}=O(1), \tag{3.5}$$

where c_p is the specific heat at constant pressure. To estimate ϵ we chose as typical values for the deep Venus atmosphere $L=1.0\times10^9$ cm, $R=1.9\times10^6$ erg (°K)$^{-1}$ gm^{-1}, $\bar{\theta}=700$K, and $\tau=4\times10^9$ sec (Gierasch *et al.*, 1970). We then find

$$\tau_D=3\times10^4 \text{ sec}, \tag{3.6}$$

$$\epsilon=0.7\times10^{-5}. \tag{3.7}$$

From the results of Gierasch *et al.* and Golitsyn we anticipate that $B\ll B_e$ when $\epsilon\ll1$. Therefore, we can take advantage of the smallness of ϵ by replacing Eq. (2.30) by

$$B_e\approx\frac{1}{\epsilon}AB^{\frac{1}{2}}. \tag{3.8}$$

Substituting for A from this equation into (2.31) we obtain

$$A^4-A_eA^3=\epsilon^2B_e^3. \tag{3.9}$$

Eq. (3.9) always has one positive real root and one negative real root. Since Eq. (3.8) requires that $A>0$, there is a unique real solution of Eqs. (3.8) and (3.9). When $\epsilon\ll1$, the solution can be found by perturbation series. We distinguish three cases:

Case (i): Statically unstable radiative states

If $A_e<0$, and $|A_e|>O(\epsilon^{\frac{1}{2}})$, then the solution is

$$\left.\begin{aligned}A&=\epsilon^{\frac{2}{3}}\frac{B_e}{|A_e|^{\frac{1}{3}}}+O(\epsilon^{\frac{4}{3}})\\ B&=\epsilon^{\frac{2}{3}}|A_e|^{\frac{2}{3}}+O(\epsilon^{\frac{4}{3}})\end{aligned}\right\}. \tag{3.10}$$

Case (ii): Radiative states near neutral stability

If $|A_e|\leqslant O(\epsilon^{\frac{1}{2}})$ the solution cannot be expressed in analytical terms, but the order of magnitude can be deduced, and the particular solution for $A_e=0$ can be found, namely

$$\left.\begin{aligned}A&=O(\epsilon^{\frac{1}{2}})\\ B&=O(\epsilon)\end{aligned}\right\}, \tag{3.11a}$$

$$\left.\begin{aligned}A&=\epsilon^{\frac{1}{2}}B_e^{\frac{3}{4}}+O(\epsilon^{\frac{3}{2}})\\ B&=\epsilon B_e^{\frac{1}{2}}+O(\epsilon^2)\end{aligned}\right\}, \quad \text{if } A_e=0. \tag{3.11b}$$

Case (iii): Statically stable radiative states

If $A_e>0$ and $|A_e|>O(\epsilon^{\frac{1}{2}})$, then the solution is

$$\left.\begin{aligned}A&=A_e+\epsilon^2\left(\frac{B_e}{A_e}\right)^3+O(\epsilon^4)\\ B&=\epsilon^2\left(\frac{B_e}{A_e}\right)^2+O(\epsilon^2)\end{aligned}\right\}. \tag{3.12}$$

Using these solutions to check our assumption that $B \ll B_e$, we find that the assumption holds unless simultaneously $A_e < 0$ and $|A_e| \geqslant O(\epsilon^{-1})$. Under such circumstances statically unstable solutions ($A<0$) can occur. This situation would only arise if the atmosphere had a very large optical depth, of the order of 10^{22} or larger for Venus. Therefore, Eqs. (3.10) to (3.12) represent essentially a complete solution for the possible states on Venus.

The above results showing three basically different kinds of equilibrium are summarized in Table 1. The order of magnitudes of the dimensionless static stability (A), the equator-to-pole temperature contrast [B, cf. Eq. (2.12)], and the velocities [$B^{\frac{1}{2}}$, cf. Eqs. (2.13) and (2.14)] are tabulated for the different order of magnitude ranges of A_e. Since B_e is necessarily of order unity, only the dependences on A_e and ϵ are shown. We see from the table that the small value of ϵ in the deep Venus atmosphere will insure that the equator-to-pole temperature contrast and the velocities are relatively small. As for the static stability, we can distinguish essentially two different situations. If the radiative state is statically stable, the static stability is virtually unaffected by the dynamics. If the radiative state is near neutral stability or statically unstable, the dynamics produces a virtually adiabatic lapse rate. In this latter case the lapse rate is subadiabatic, but only by very small amounts. Only in this case is the greenhouse effect strong enough to cause the high surface temperatures on Venus, and therefore this is the case we expect to occur on Venus.

Our qualitative results can be explained in a straightforward physical way. Consider first the case of a statically stable radiative state. Because of the very long radiative time constant and very short dynamical time constant, the meridional dynamical fluxes are much more efficient than differential solar heating, and the equator-to-pole temperature contrast is almost wiped out. Since the small meridional temperature contrast makes vertical motions very inefficient at transporting heat upward, the dynamics hardly modifies the static stability of the radiative state. Next consider the changes as the static stability decreases. Now the entropy difference between the poleward and equatorward branches of the Hadley cell becomes smaller and the meridional motions become less efficient at transporting heat. Consequently, the meridional temperature contrast increases and the vertical dynamical fluxes became more efficient. When the static stability approaches zero there is an order of magnitude decrease in the efficiency of the meridional dynamical flux and an order of magnitude increase in the efficiency of the vertical dynamical flux. Thus it is very difficult to get superadiabatic lapse rates and the equator-to-pole temperature contrast is relatively much larger.

TABLE 1. Parameter dependences of equilibrium atmospheric properties when $\epsilon \ll 1$.

	Radiative-dynamical state		
Radiative state	Static stability	Temperature contrast	Velocity magnitude
Statically unstable $-\epsilon^{-1} < A_e < -\epsilon^{\frac{1}{2}}$	$\epsilon^{\frac{2}{3}}\lvert A_e\rvert^{-\frac{1}{3}}$	$\epsilon^{\frac{2}{3}}\lvert A_e\rvert^{\frac{2}{3}}$	$\epsilon^{\frac{1}{3}}\lvert A_e\rvert^{\frac{1}{3}}$
Near neutral stability $-\epsilon^{\frac{1}{2}} \leqslant A_e \leqslant \epsilon^{\frac{1}{2}}$	$\epsilon^{\frac{1}{2}}$	ϵ	$\epsilon^{\frac{1}{2}}$
Statically stable $\epsilon^{\frac{1}{2}} < A_e$	A_e	$\epsilon^2 A_e^{-2}$	ϵA_e^{-1}

4. The adjustment problem

We can also use the approach of Section 2 to study how the deep atmosphere reaches an equilibrium, given an arbitrary initial condition. Such a calculation will reveal the important time scales governing variations in the temperature structure, and show how long numerical integrations seeking equilibrium solutions have to be carried forward in simulated time. To derive the time-dependent equations, we relax the assumption that the solution is steady, and replace (2.4) by

$$\frac{\partial\theta}{\partial t}+\frac{\partial}{\partial y}(v\theta)+\frac{\partial}{\partial p}(\omega\theta)=\frac{\theta_e-\theta}{\tau}. \tag{4.1}$$

We will continue to neglect time derivatives in the equation of motion, i.e., we will assume that the time required for the motions to come into balance with the temperature field is much less than the time required for the temperature field to reach equilibrium. We will test this assumption *a posteriori.*

We derive time-dependent equations for A and B from Eq. (4.1) in a manner identical to that used for deriving the equilibrium equations for A and B in Section 2. The only difference is that θ is now allowed to vary in time. The resulting equations are

$$\frac{dB}{dt'}+B-B_e=-\frac{1}{\epsilon}AB^{\frac{1}{2}}, \tag{4.2}$$

$$\frac{dA}{dt'}+A-A_e=\frac{1}{\epsilon}B^{\frac{3}{2}}, \tag{4.3}$$

where t' is a dimensionless time variable,

$$t'=\frac{t}{\tau}. \tag{4.4}$$

The time scales contained in the solution of Eqs. (4.2) and (4.3) depend on the magnitudes of A and B. As we saw in the preceding section, these magnitudes depend on A_e and therefore different time scales are appropriate for different ranges of A_e.

Case (i): Statically unstable radiative states

The equilibrium solution is given by Eq. (3.10), and we define new variables,

$$a=\frac{A}{\epsilon^{\frac{2}{3}}}=\mathrm{O}(1), \tag{4.5}$$

$$b=\frac{B}{\epsilon^{\frac{2}{3}}}=\mathrm{O}(1). \tag{4.6}$$

Substituting these expressions into (4.2) and (4.3), we obtain

$$\epsilon^{\frac{2}{3}}\left(\frac{db}{dt'}+b\right)=B_e-ab^{\frac{1}{2}}, \tag{4.7}$$

$$\epsilon^{\frac{2}{3}}\left(\frac{da}{dt'}+a\right)=b^{\frac{3}{2}}-|A_e|. \tag{4.8}$$

Inspection shows that there will be two time scales in the adjustment:

$$t'=\begin{Bmatrix}\mathrm{O}(\epsilon^{\frac{2}{3}})\\ \mathrm{O}(1)\end{Bmatrix}. \tag{4.9}$$

The second time scale is the radiative relaxation time scale, and the first is essentially an advective time scale (see below). To study changes on the advective time scale, we define a new time variable,

$$t'=\epsilon^{\frac{2}{3}}t^*. \tag{4.10}$$

Eqs. (4.7) and (4.8) now become

$$\frac{db}{dt^*}=B_e-ab^{\frac{1}{2}}+\mathrm{O}(\epsilon^{\frac{2}{3}}), \tag{4.11}$$

$$\frac{da}{dt^*}=b^{\frac{3}{2}}-|A_e|+\mathrm{O}(\epsilon^{\frac{2}{3}}). \tag{4.12}$$

Since the pair of equations is second order in t^*, an arbitrary initial condition will adjust to equilibrium within an error of order $\epsilon^{\frac{2}{3}}$ on the advective time scale. If we linearize Eqs. (4.11) and (4.12) by assuming small perturbations from equilibrium, and assume that the perturbations are of the form $e^{\sigma t^*}$, we find

$$\sigma=-\frac{B_c}{4|A_e|^{\frac{2}{3}}}\left[1\pm\left(1-\frac{24A_e^2}{B_e^2}\right)^{\frac{1}{2}}\right]. \tag{4.13}$$

Thus the perturbations are always damped, and they will also oscillate if $B_e<2\sqrt{6}|A_e|$. For example, if we arbitrarily choose $B_e=|A_e|=1$, we find

$$\sigma=-0.25\pm1.2i. \tag{4.14}$$

Using the values of τ and ϵ given previously, we find that this solution oscillates with a period of 8×10^6 sec, and is damped by a factor $1/e$ in 6×10^6 sec. The remaining deviations from equilibrium, of order $e^{\frac{2}{3}}$, are finally removed on the long radiative time scale. For all practical purposes a complete adjustment is attained on the advective time scale. If we take into account the A_e dependence in Eq. (4.13), this time scale is

$$t'=\begin{Bmatrix}\mathrm{O}(\epsilon^{\frac{2}{3}}|A_e|^{\frac{2}{3}}), & \text{if } |A_e|\geqslant\mathrm{O}(1)\\ \mathrm{O}(\epsilon^{\frac{2}{3}}|A_e|^{-\frac{4}{3}}), & \text{if } |A_e|<\mathrm{O}(1)\end{Bmatrix}. \tag{4.15}$$

Case (ii): Radiative states near neutral stability

Now solution (3.11) is appropriate, and a and b must be defined differently if they are to be of order unity. We redefine

$$a=\frac{A}{\epsilon^{\frac{1}{2}}}=\mathrm{O}(1), \tag{4.16}$$

$$b=\frac{B}{\epsilon}=\mathrm{O}(1), \tag{4.17}$$

$$a_e=\frac{A_e}{\epsilon^{\frac{1}{2}}}=\mathrm{O}(1). \tag{4.18}$$

Substituting these into (4.2) and (4.3), we obtain

$$\epsilon\left(\frac{db}{dt'}+b\right)-B_e=-ab^{\frac{1}{2}}, \tag{4.19}$$

$$\frac{da}{dt'}+a-a_e=b^{\frac{3}{2}}. \tag{4.20}$$

By inspection we see that again there are two time scales in the adjustment, but now they are

$$t'=\begin{Bmatrix}\mathrm{O}(\epsilon)\\ \mathrm{O}(1)\end{Bmatrix}. \tag{4.21}$$

These correspond to the dimensional time scales τ_D and τ.

To determine the behavior on the short time scale, we redefine

$$t'=\epsilon t^*. \tag{4.22}$$

We substitute this definition into (4.19) and (4.20), let $\epsilon\rightarrow0$, and obtain

$$\frac{db}{dt^*}-B_e=-ab^{\frac{1}{2}}, \tag{4.23}$$

$$\frac{da}{dt^*}=0. \tag{4.24}$$

If we specify as initial conditions

$$\left.\begin{matrix}b=b_0\\ a=a_0\end{matrix}\right\},\quad \text{at } t=0, \tag{4.25}$$

we can implicitly integrate Eqs. (4.23) and (4.24) to find

$$a=a_0 \tag{4.26}$$

$$\frac{a_0b^{\frac{1}{2}}-B_e}{a_0b_0^{\frac{1}{2}}-B_e}\exp\left[\frac{a_0}{B_e}(b^{\frac{1}{2}}-b_0^{\frac{1}{2}})\right]=\exp\left(-\frac{a_0{}^2t^*}{2B_e}\right). \tag{4.27}$$

Therefore, a is unchanged over the short time scale, while b evolves from its initial value to a quasi-equilibrium value B_e^2/a_0^2.

To determine the behavior over the long time scale, $t'=O(1)$, we let $\epsilon\rightarrow 0$ in Eqs. (4.19) and (4.20). The latter equation is unchanged, while the former reduces to the quasi-equilibrium relation

$$b^{\frac{1}{2}}=\frac{B_e}{a}. \tag{4.28}$$

Substituting this into (4.20), we find

$$\frac{da}{dt'}+a-a_e=\frac{B_e{}^3}{a^3}. \tag{4.29}$$

Therefore, a evolves from its initial value to its equilibrium value over the long time scale, and the quasi-equilibrium value of b evolves simultaneously. We can solve Eq. (4.29) in the special case $a_e=0$. If we apply as an initial condition (4.26) so that the solutions for the short and long time scales join smoothly, we obtain

$$a=[a_0e^{-4t'}+B_e^3(1-e^{-4t'})]^{\frac{1}{4}}, \tag{4.30}$$

$$b=B_e^2[a_0^4e^{-4t'}+B_e^3(1-e^{-4't})]^{-\frac{1}{2}}. \tag{4.31}$$

Therefore, a complete adjustment to equilibrium is only attained on the long radiative time scale.

Case (iii): Statically stable radiative states

Now the appropriate equilibrium solution is given by Eq. (3.12). Thus we redefine

$$a=\frac{A}{A_e}=O(1), \tag{4.32}$$

$$b=\frac{B}{\epsilon^2}=O(1). \tag{4.33}$$

Substituting these into (4.2) and (4.3), we find

$$\epsilon^2\left(\frac{db}{dt'}+b\right)=B_e-A_eab^{\frac{1}{2}}, \tag{4.34}$$

$$A_e\left(\frac{da}{dt'}+a-1\right)=\epsilon^2b^{\frac{3}{2}}. \tag{4.35}$$

By inspection we see that now the relevant time scales are

$$t'=\begin{Bmatrix} O(\epsilon^2) \\ O(1) \end{Bmatrix}. \tag{4.36}$$

To find the time dependence on the short time scale we redefine

$$t'=\epsilon^2t^*, \tag{4.37}$$

substitute for t' in (4.34) and (4.35), and let $\epsilon\rightarrow 0$. We find

$$\frac{db}{dt^*}=B_e-A_eab^{\frac{1}{2}}, \tag{4.38}$$

$$\frac{da}{dt^*}=0. \tag{4.39}$$

These equations have the same form as Eqs. (4.23) and (4.24) and their solution is similar. We find that a retains its initial value on the short time scale, while b is given by Eq. (4.27) with a_0 replaced by a_0A_e. Again b attains a quasi-equilibrium value on the short time scale.

To find the time dependence on the long term scale we let $\epsilon\rightarrow 0$ in Eqs. (4.34) and (4.35), and obtain

$$B_e=A_eab^{\frac{1}{2}}, \tag{4.40}$$

$$\frac{da}{dt'}+a-1=0. \tag{4.41}$$

The solution is

$$a=1+(a_0-1)e^{-t'}, \tag{4.42}$$

$$b=\frac{B_e{}^2}{A_e{}^2}[1+(a_0-1)e^{-t'}]^{-2}. \tag{4.43}$$

Again a complete adjustment to equilibrium is only attained over the long radiative time scale.

Now we check our assumption that the time required for the motions to come into balance with the temperature field is much less than the time required for the temperature field to reach equilibrium. The former time scale is the advective time scale, L/v. Referring to Eq. (2.13) for the magnitude of v, we define

$$\tau_A\equiv\frac{L}{(R\hat{\theta}B)^{\frac{1}{2}}}=\frac{\epsilon\tau}{B^{\frac{1}{2}}}=O\left(\frac{L}{v}\right). \tag{4.44}$$

Substituting into (4.44) our equilibrium solutions for B [Eqs. (3.10)–(3.12)], we obtain

$$\tau_A=O\left(\frac{\epsilon^{\frac{2}{3}}\tau}{|A_e|^{\frac{1}{3}}}\right),\quad \text{if}\ \begin{Bmatrix} A_e<0 \\ |A_e|>O(\epsilon^{\frac{1}{2}}) \end{Bmatrix}, \tag{4.45}$$

$$\tau_A=O(\epsilon^{\frac{1}{2}}\tau),\quad \text{if}\ |A_e|\leqslant O(\epsilon^{\frac{1}{2}}), \tag{4.46}$$

$$\tau_A=O(A_e\tau),\quad \text{if}\ \begin{Bmatrix} A_e>0 \\ |A_e|>O(\epsilon^{\frac{1}{2}}) \end{Bmatrix}. \tag{4.47}$$

Comparing these advective time scales with the time scales we found for the adjustment of the temperature field to equilibrium, we see that there is only one situation where the former exceeds the latter—when simultaneously $A_e>0$ and $|A_e|>O(1)$. In this case we expect a complete adjustment to require the longer advective time scale. Also there is one special case when the time scales are the same—when $|A_e|=O(1)$, regardless of the sign of A_e. In this case our solutions for the adjustment given above will be quantitatively inaccurate, but the basic time scale of the adjustment process will not be changed. In all other cases our original assumption is valid and our solutions given above for the adjustment time scale are accurate.

Our results for the adjustment time scale when $\epsilon\ll 1$ are summarized in Table 2. This table shows the order of magnitude of the dimensionless adjustment time (t/τ) for the different order of magnitude ranges of A_e. For neutral or stable radiative states, the adjustment occurs on the radiative relaxation time scale or longer. For unstable radiative states the adjustment occurs on an advective time scale which reaches a minimum when $A_e<0$ and $|A_e|=O(1)$. This advective time scale can be considerably less than the radiative time scale if ϵ is sufficiently small.

Finally we note that the mean potential temperature $\bar{\theta}$ adjusts to equilibrium independently of A and B. Combining Eq. (2.20) with Eq. (4.1) averaged over all y and p, we find

$$\frac{\partial\bar{\theta}}{\partial t}=\frac{\bar{\theta}_e-\bar{\theta}}{\tau}. \tag{4.48}$$

Therefore, $\bar{\theta}$ always relaxes to its equilibrium value with the radiative time scale. This adjustment is of limited interest since it does not affect the atmospheric structure or dynamics.

5. Discussion

Our analysis in Sections 2–4 is applicable to atmospheres for which the following assumptions hold: (i) the thermal inertia is large enough that diurnal effects are negligible and $\epsilon\ll 1$; (ii) the rotation rate is slow enough that the dominant large-scale circulation will be an overturning Hadley cell; and (iii) the motions are driven directly by local radiative heating. Our results show that there are three qualitatively different states which such atmospheres can have, depending on the strength of the radiative heating, as measured by the static stability A_e of the radiative state. Table 1 shows the dependence of the important dimensionless quantities on ϵ and A_e.

All the states are characterized by small meridional temperature contrasts and velocities. The lapse rates resemble those produced by small-scale convection. If the radiative state is stable the lapse rate is essentially unchanged from that in the radiative state, and if the radiative state is unstable the lapse rate is nearly adiabatic. However, in the latter case the lapse rate is slightly subadiabatic, so small-scale convection will not occur, at least in a mean sense. The magnitudes given in Table 1 refer only to mean quantities, and we cannot exclude the possibility that local values will vary substantially in magnitude. Such variations are especially likely above the lowest scale height because of the decrease of pressure and density. Conditions in these regions have a relatively small effect on mean atmospheric quantities.

TABLE 2. Parameter dependences of the time necessary to reach radiative-dynamical equilibrium when $\epsilon\ll 1$.

Radiative state	Adjustment time
$-\epsilon^{-1}<A_e<-1$	$\epsilon^{\frac{2}{3}}\|A_e\|^{\frac{2}{3}}$
$-1<A_e<-\epsilon^{\frac{1}{2}}$	$\epsilon^{\frac{2}{3}}\|A_e\|^{-4/3}$
$-\epsilon^{\frac{1}{2}}<A_e<1$	1
$1<A_e$	A_e

In view of our discussion in Section 1, the assumptions made in our analysis are reasonable ones for the deep Venus atmosphere. If we use the same values of the parameters specified in the preceding sections as typical of Venus, then the dimensionless magnitudes given in Table 1 can be converted to dimensional magnitudes by multiplying by the following units: 50K km^{-1} for the static stability, 700K for the meridional temperature contrast, 400 m sec^{-1} for the horizontal velocity, and 60 cm sec^{-1} for the vertical velocity. Since $\epsilon=O(10^{-5})$ for Venus, the only states consistent with the observations are those corresponding to $A_e\leqslant-\epsilon^{\frac{1}{2}}$. These states have just the properties anticipated by our discussion in Section 1. The lapse rate is very close to the adiabatic lapse rate, but sufficiently far from it that the dynamical transports reduce the meridional temperature contrast to very small values. Furthermore, they are the only states which have horizontal velocities as large as a few meters per second. With $\epsilon=0.7\times10^{-5}$, if we arbitrarily specify that the static stability of the radiative state is -1K km^{-1} ($A_e=-0.02$), we calculate the following magnitudes for the equilibrium state: a mean static stability of $+0.07$K km^{-1}, an equator-to-pole temperature contrast of 0.02K, a horizontal velocity of 2 m sec^{-1}, and a vertical velocity of 0.4 cm sec^{-1}. All these values are consistent with our current knowledge of the deep Venus atmosphere. We conclude that the hypothesis that the deep circulation is basically a Hadley cell, with the temperature structure determined by the radiative and large-scale dynamical fluxes, is a viable one. We note that the velocities are not strongly dependent on A_e and therefore the above velocity estimates are fairly firm ones. The equator-to-pole temperature contrast is more strongly dependent on A_e, but it would be reasonable to conclude, for example, that the contrast is unlikely to exceed 0.1K.

Our results for the adjustment time, summarized in Table 2, are particularly useful in evaluating the numerical calculations of the Venus circulation. The dimensionless time scales in this table may be converted to dimensional times by multiplying by the radiative relaxation time τ which is about 100 earth years for Venus. Turikov and Chalikov (1971) neglected long-wave radiation and assumed that the radiative response of the deep atmosphere was instantaneous, i.e., $\tau=0$. Therefore, their results are not very meaningful for Venus. Hess (1968), Sasamori (1971), and de Rivas (1973) all studied situations with $A_e>0$, but only integrated for periods of about 200 earth days. Since the adjustment time scale for the deep atmosphere in these cases is of order τ or larger, their solutions only describe quasi-equilibrium states and would have continued to evolve slowly if the integrations had been continued. However, their solutions may have reached equilibrium in the upper parts of the atmosphere where the effective value of τ is much less.

Extending these numerical calculations with $A_e>0$ would be a formidable problem because of the computer time required. However, our results suggest that calculations with $A_e<-\epsilon^{\frac{1}{2}}$ are more relevant for Venus. For these states the adjustment time is an advective time scale, which can be as small as 20 earth days (if $A_e\approx-1$). It would be relatively easy to explore these states numerically.

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